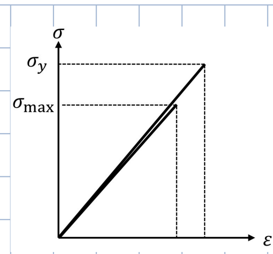
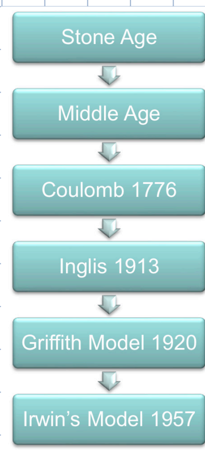
- Presence of defects modifies local stress field in material
→ Elastic stress analysis assumes perfectly homogenous & flawless materials
↳ not suitable for high-strength material design



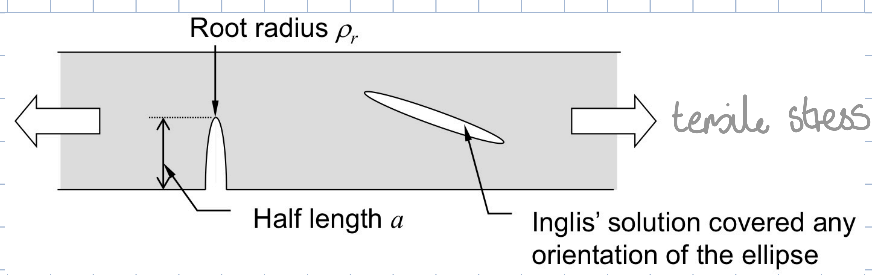
- When a crack reaches a critical length it can propagate catastrophically through structure
↳ this can happen for below yield stress of material

- Fracture mechanics refers to vital specialisation within solid mechanics in which the presence of flaw assumed.

→ Relationships established between material's inherent resistance to crack growth, crack length and far-field stress that results in failure.



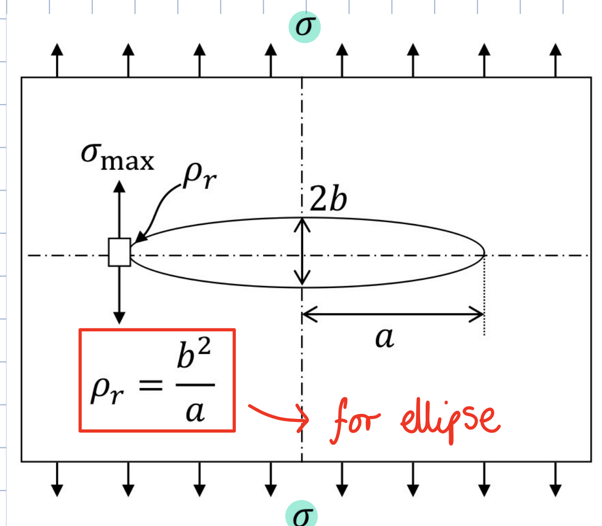
Elliptical Notch - Inglis Solution :



$$\boxed{\sigma_{max} = R\left(1 + 2\sqrt{\frac{a}{\rho_r}}\right)}$$

'R' depends on orientation of notch & details of stress field etc.

↳ for an elliptical notch normal to direction of uniform (far field) tensile stress σ, in an infinite plate :



$R = \sigma$, $\therefore$ $\boxed{\sigma_{max} = \sigma\left(1 + 2\sqrt{\frac{a}{\rho_r}}\right)}$

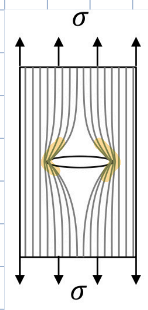
for circular hole, $a = b$ $\therefore$ $\sigma_{max} = 3\sigma$

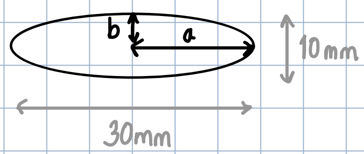
Ratio $\frac{\text{max stress}}{\text{nominal applied stress}} = \boxed{\frac{\sigma_{max}}{\sigma} = k_t}$

↳ k_t = stress conc. factor

examples : same ellipse (30 mm long, 10 mm wide, but 90° rotated)

↳ a & b swap

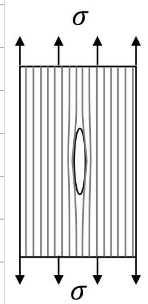


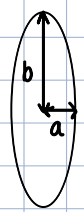


$$\rho_r = \frac{b^2}{a} = \frac{5^2}{15}$$

$$k_t = 1 + 2\sqrt{\frac{15}{25/15}} = 7$$ → much higher

→ 'a' always horizontal





$$\rho_r = \frac{b^2}{a} = \frac{15^2}{5}$$

$$k_t = 1 + 2\sqrt{\frac{15}{45}} = 1.67$$

REMEMBER a & b are HALF THE LENGTH & WIDTH

→ as $b \downarrow$, $\rho_r \to 0$ (sharp), $a/\rho_r >> 1$

↳ $\boxed{\sigma_{tip} \approx 2\sigma\sqrt{\frac{a}{\rho_r}}}$ Sharp notches

→ this result would predict infinite stress at perfectly sharp crack tip

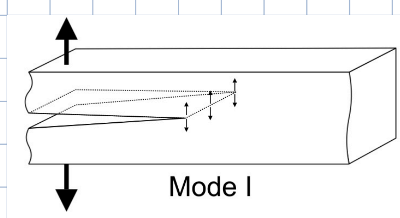
↳ predicts 0 strength

↳ non-physical

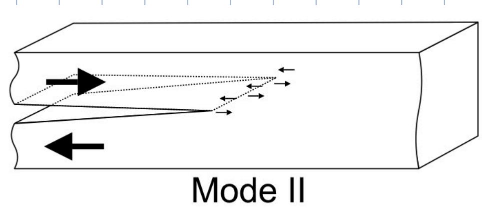
↳ in reality, material generally undergoes local yielding which blunts crack tip

Stress Intensity Factor :

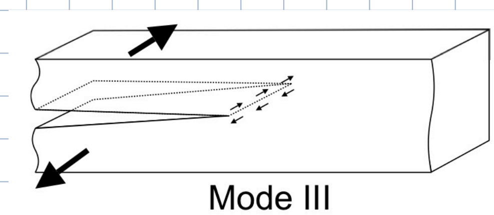
Stress at tip can undergo 3 modes of loading :



Tensile Opening



Shear Sliding



Tearing

→ Many real fractures are mixture but mode 1 & 2 tend to dominate

- Irwin generalised Inglis equation for stress field ahead of crack tip :

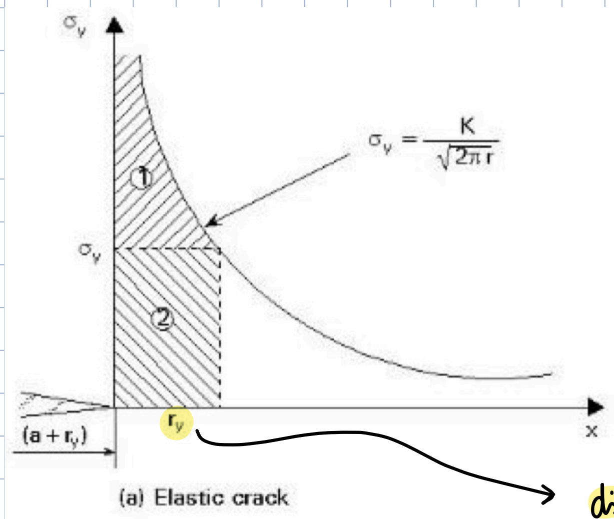
$$\boxed{\frac{1}{\sqrt{2\pi r}}}\boxed{\sigma\sqrt{\pi a}}\boxed{f(\theta)}$$

singularity
where r = dis from
tip to desired point

stress intensity factor , K
where σ = far-field stress

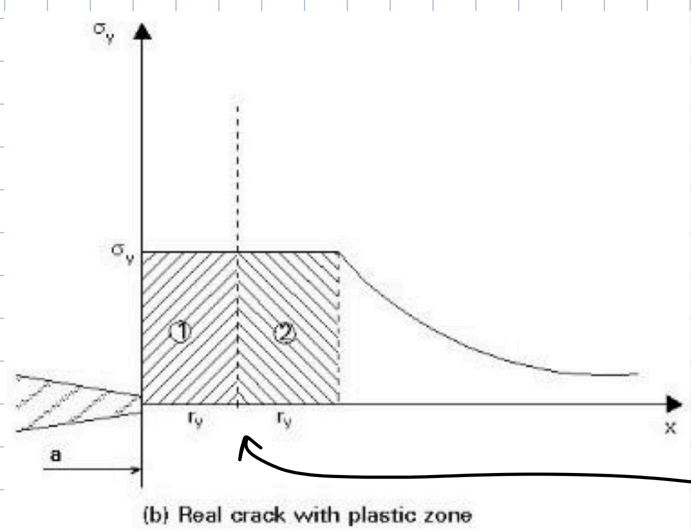
constant for specific crack geometry &
applied stress

angular dependance - orientation to far-field
stress



(a) Elastic crack

distance from crack tip where stress equal
to material yield strength



(b) Real crack with plastic zone

Plastic zone, where large part of
energy absorbed during fracture, $= 2r_y$

- For initial case of infinite plate with internal crack length 2a , subjected to uniform
stresses :

$$\boxed{K = \sigma\sqrt{\pi a}}$$

→ In general, K highly dependent on geometry of cracked body:

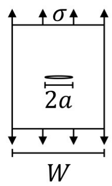
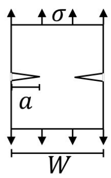
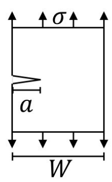
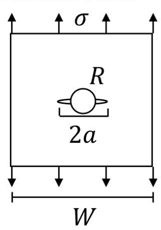
$$\boxed{K = \beta \sigma \sqrt{\pi a}}$$

↳ β determined by geometric config.

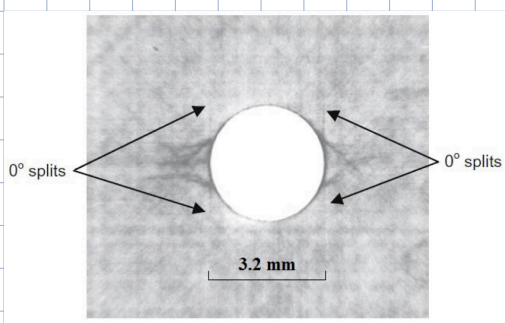
- Critical K when fracture occurs = fracture toughness, K_c:

$$\boxed{K_c = \beta \sigma_s \sqrt{\pi a}}$$

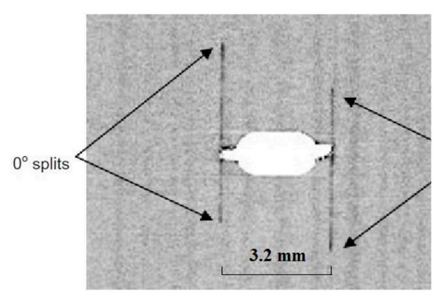
Common Geometries & Corresponding β:

Centre Cracked Plate		**Double Edge Notched Plate**	
	$K_I = \beta\sigma\sqrt{\pi a}$ $\beta = \sqrt{sec\frac{\pi a}{W}}$		$K_I = \beta\sigma\sqrt{\pi a}$ $\beta = 1.12$ for small cracks
Single Edge Notched Plate		**Cracked Hole**	
	$K_I = \beta\sigma\sqrt{\pi a}$ $\beta = 1.12$ for small cracks		$K_I = \beta\sigma\sqrt{\pi a}$ $\beta = f(R, a, W)$

- In composites, sharp notches are stronger because they have longer 0° splits which blunt stress concentrations.



(a) Open-hole specimen



(b) Centre-notched specimen